correlation:

$$\xi^* = (a\chi_0 + b)(1 + 0.245 Re_0^{1/2})^P \qquad (11)$$

where

$$a = 0.504(2.0 + A_0)^{0.440}$$
$$b = 0.177(1.5 + A_0)^{0.107}$$

and

$$P = 1.29 - 0.815/\chi_0 \quad \chi_0 \geq 1$$
$$= 0.462 + 0.013/\chi_0 \quad \chi_0 \leq 1$$

This relationship for ξ^* agrees with the numerical results obtained within 7%.

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Thermal Mission Analysis for the Lunar Module

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THERMAL mission analysis for the Apollo Lunar Module (LM) uses the Grumman Thermal Analyzer (GTA), a Fortran program for the IBM 360/75. This program operates on basic LM ascent-stage and descent-stage thermal networks, which have ~700 and 400 lumped-mass nodes, respectively; most of these nodes are conductively and radiatively coupled with several other nodes. A mission consists of numerous timeline events, among which the thermal events (window shade deployments, hatch openings, rocket engine firings, electronic equipment actuations, etc.) are called parameter changes, since they are accomplished by defining new values for some of the basic parameters: conductors, radiators, and heat sources. In general, a thermal network is defined at the time of a parameter change, and the GTA calculates the transient thermal response until the next parameter change.

For a typical Apollo mission, ~5000 data cards for these parameter changes are used for the LM ascent stage. Since one mispunched or misplaced card can invalidate a computer run, thermal mission analysis is confronted with a serious data handling problem. One approach to reduce the amount of data to manageable levels was to reduce the size of the thermal networks.[1] These simplified networks were found to be useful tools for rapid parametric design analyses. Both simplified and basic networks have been verified by full-scale thermal vacuum testing (for earth orbital and lunar landing missions) as well as by previous Apollo flights. For manned missions, however, the analyses are performed with the basic thermal networks, since a much greater level of detail is required. Thus, it became necessary to take another approach —namely to automate and link the generation, manipulation, and interpretation of data. This approach is the one described in this Note.

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Thermal Mission Analysis

Thermal mission analysis is accomplished in three steps: 1) the generation of detailed mission timeline data by T-CUP (Timeline and Consumable Usage Program),[2] 2) scanning this data for thermal significance by TSP (Timeline Scanning Program), and 3) the determination of thermal mission constraints by the GTA.

The T-CUP details every planned functional change with respect to time for the anticipated mission. Thus, timeline data concerning astronaut activity schedules, consumable budgets and other mission related data are available (nearly in real time) to prepare and modify the various LM mission timelines (both nominal and contingency) and to determine the usage of expendables. To establish total electrical power consumption, T-CUP generates the usage mode of every equipment on the LM. This detailed mode history of 125 pieces of equipment as well as relevant astronaut/spacecraft activity and consumables are recorded on an output tape (Thermdmp).

The TSP searches the T-CUP output tape for thermal events such as rocket engine firing duration, cabin pressurization, fan operation, glycol coolant loop operation, and equipment on-off switching related to heat dissipations in order to determine times for parameter changes for the GTA program. With these times established, the TSP time-averages the equipment heat loads for the duration of the parameters and transmits this information as well as other thermal data (i.e., rocket engine firing, etc.) to the GTA by means of a compatible output tape. Some equipments have as many as 8 modes of operation and a particular mode may yield heat inputs to as many as 13 network nodes. Also, a particular network node may receive heat input from several different equipments. For a typical mission, over 3000 data cards were required previously to represent all equipment heat input changes. The TSP yields even more detailed information on its Thermdmp output tape (totally eliminating keypunch and card handling errors) as well as defining function control variables (described below) for other thermally significant events.

Thermal mission constraints can be found from the output of the GTA. The thermal data of the TSP output tape together with the basic network representation and typically 30 cards (primarily launch conditions, thermal blanket insulation effectiveness, and spacecraft orientation with respect to the sun-earth-moon at various mission times) serve as input to the GTA. The printed output of lumped-mass-node temperature histories and vehicle heat fluxes, as well as CRT data display curves, can be used to determine if thermal specification limits have been exceeded.

To improve the data handling capability of the GTA, it was recognized that a complex thermal network in principle is identical to a simple analytical model that is suitable for hand calculations. However, with the more complex model it is usually necessary to do many operations for which one suffices in the simple model. Since many thermal events require fixed blocks of data (i.e., combinations of temperatures, masses, conductors, radiators, and heat sources) they can be handled with subroutines triggered by only one variable. With the addition of data handling subroutines to the GTA, it is now possible to control about 25 thermal events for the ascent stage network and 20 events for the descent stage network through single function control variables. (For an ascent stage rocket engine control variable specified as "327," the GTA subroutines will consider this to be a 327-sec engine firing, separate the LM ascent stage from the descent stage, disconnect the coolant loop flow to the descent stage, rupture the ascent engine diaphragm, and force the ascent engine boundary nodes to a temperature profile consistent with a 327-sec firing. This normally corresponds to about 200 cards. Similarly, another firing would again substitute one function control card for the conventional 200.) These function control variables control events such as LM orientations, glycol coolant loop operation, initial temperatures, cabin pressurization, consumable usage, etc. This technique substituted less than 30 function control variable cards for about 2000 conventional data cards. Some function control variables are defined automatically by the TSP, others are defined manually.

Concluding Remarks

A close to real time thermal mission analysis capability was developed for the Apollo lunar landing missions. This capability will also be needed for later Apollo and Apollo Applications missions and enable quick determination of flight constraints both before and after lift-off. Turnaround time of a mission analysis typically was of the order of two months; today it can be a matter of hours.

The automated methods developed allow the use of as much automation as desired; one can perform a thermal mission analysis with over 5000 conventional data cards (the data handling subroutines in the GTA remain inactive until called) or combine a totally automated analysis (~30 cards) with conventional card input for a localized area. The automation does not replace engineering judgment, but it frees the engineer of much tedious work and gives him more time to analyze results and update the basic network. The thermal mission analysis is only as valid as the thermal network and the data contained in the various subroutines.

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A Least-Squares Estimate of the Attitude of a Satellite

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THE attitude reconstitution of a nonspin stabilized spacecraft is often reduced to finding a transformation that brings a set of measured vectors into the best coincidence with a corresponding set of model vectors. The solution of this problem for a set of n vectors ($n \geq 2$), optimal in the least squares sense, was given by Farrell and Stuelpnagel.[1] The rather lengthy computation hidden behind the general method outlined in Ref. 1 can be avoided in the special case of two noncolinear measured vectors, V_1 and V_2, and two noncolinear image vectors, W_1 and W_2, respectively. This had been pointed out by P. B. Davenport,[2] but with the condition $\|V_1\| = \|V_2\|$ and $\|W_1\| = \|W_2\|$. We show below that without this condition, a general solution is obtained by a very short algorithm. The method was developed for and applied to the attitude reconstitution of the ESRO I spacecraft, measuring the solar and magnetic vectors in the body axes.

One assumes in the following that the vectors are arranged as column vectors and the transpose of a matrix is denoted by a superscript t; to indicate the trace of a matrix, one employs the abbreviation tr. We construct a 3×2 matrix A by writing V_i in the i-th column; a similar matrix B is built by the vectors W_i. The rotation matrix R one has to deduce, transforms B into A. But the matrix $C = A - RB$ is normally not a null matrix, because of errors that may be present in B as well as in A.

The sum of all elements, raised to the square of C is a function of R and is given by

$$e(R) = tr(C^TC)$$

The matrix R, yielding the minimum of $e(R)$ is the best solution to our problem in the least squares sense. By substituting $A - RB$ for C in the preceding expression, it becomes obvious that min $e(R)$ is equivalent to

$$\max \operatorname{tr}(A^TRB)$$

because the other terms are independent of R.

To find this maximum, one introduces a rotation matrix P, which turns the vector $V_1\mathbf{X}V_2$ onto a fixed axis, and a similar rotation matrix Q, rotating $W_1\mathbf{X}W_2$ onto the same axis in the same direction. For simplicity, but without loss of generality, the z-axis is chosen for the fixed axis in the arguments following. The matrices P and Q are not unique. The matrix P can be constructed by superposing the vectors $V_1\mathbf{X}\ (V_2\mathbf{X}V_1)$, V_1, and $V_2\mathbf{X}V_1$ in the successive rows after normalization. The matrix Q can be built in an analogous way by using W_1 and W_2. The residue is now modified as follows:

$$e'(R) = \operatorname{tr}(PA - PRQ^TQB)^T(PA - PRQ^TQB)$$

which does not affect the solution, for

$$\max \operatorname{tr}(A^TP^TPRQ^TQB) = \max \operatorname{tr}(A^TRB) \qquad (1)$$

The maximum can be computed by considering $M = PRQ^T$ as the unknown matrix, instead of R.

It remains to be proven that M is a rotation around the axis onto which the cross products of the vectors V_i and W_i are rotated. The symbol K denotes the 3×3 matrix obtained from QBA^TP^T. From the assumption for P and Q, one knows that the elements k_{i3} and k_{3i} (with $i = 1, 2, 3$) of K have to be zero.

Writing the element of M as m_{ij}, one restates the problem as

$$\begin{aligned}\max_R \operatorname{tr}(A^TRB) &= \max_M \operatorname{tr}(MK) \\ &= \max_{m_{ij}} (m_{11}k_{11} + m_{12}k_{21} + m_{21}k_{12} + m_{22}k_{22}) \qquad (2) \\ &= \max_{m_{ij}} F_0(m_{ij})\end{aligned}$$

with the constraints on the m_{ij}, which are

$$F_1 = \sum_{i=1}^{3} m_{1i}^2 - 1 = 0;\ F_2 = \sum_{i=1}^{3} m_{2i}^2 - 1 = 0 \qquad (3)$$

and

$$F_3 = \sum_{i=1}^{3} m_{1i}m_{2i} = 0 \qquad (4)$$

The given constrained extremum problem can be solved by introducing the Lagrange multipliers λ_a, λ_b and λ_c and building the expression that follows:

$$F = F_0 + \lambda_aF_1 + \lambda_bF_2 + \lambda_cF_3$$

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